A Nützliche Formeln

Metrischer Tensor: $g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

4-er Vektoren: $a^\mu = (a_0, \vec{a}) \quad a_\mu = g_{\mu\nu} a^\nu = (a_0, -\vec{a})$

Lorentz-invariant: $a \cdot b = a_\mu b^\mu = a^\mu b_\mu = a_0 b_0 - \vec{a}\vec{b}$

$a^2 = a_\mu a^\mu = a_0^2 - \vec{a}^2$

4-er Impuls: $p^\mu = (E, \vec{p})\,, \quad p^2 = E^2 - \vec{p}^2 = m^2$

4-er Ableitung: $\partial^\mu = \frac{\partial}{\partial x_\mu} = (\partial_t, -\nabla)$

Impuls-Operator: $p^\mu = i\partial^\mu = (i\partial_t, -i\nabla)$

$p_\mu p^\mu = -\partial_\mu \partial^\mu = -(\partial_t^2, -\nabla^2) = -\Box$

Pauli Matrizen: $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z), \quad [\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k, \quad \{\sigma_i, \sigma_j\} = 2\delta_{ij} I$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Dirac Gleichung: $(i\gamma^\mu \partial_\mu - m)\,\psi = 0$

γ- Matrizen (4x4): $\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Anti-Kommutator: $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$

$\gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0$

wie 4-er Vektor: $\gamma_\mu = g_{\mu\nu}\gamma^\nu$

hermitesch konjugiert: $\gamma^{0\dagger} = \gamma^0 \quad \gamma^{k\dagger} = -\gamma^k \quad k = 1, 2, 3$

$\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0 \quad \gamma^{5\dagger} = \gamma^5$

Quadrate: $(\gamma^0)^2 = -(\gamma^k)^2 = (\gamma^5)^2 = \mathbb{1}$

Dagger: $\not{a} \equiv \gamma^\mu a_\mu = \gamma^0 a^0 - \vec{\gamma}\vec{a}$

$\gamma_\mu \not{a} \gamma^\mu = -2\not{a} \quad \gamma_\mu \not{a}\not{b}\gamma^\mu = 4a^\nu b_\nu$

Spur Theoreme

$Tr(A) = \Sigma_i A_{ii}$ Diagonal-Elemente

$Tr(ABC) = Tr(BCA)$

$Tr\,\mathbb{1} = 4$ ($\mathbb{1}$ = 4x4 Matrix)

$Tr\,\gamma_\mu = 0 \quad Tr\,\gamma_5 = 0$

$Tr(\text{ungerade} \cdot \gamma\text{- Matrizen}) = 0$

$Tr(\gamma_\mu \gamma_\nu) = 4g_{\mu\nu}$

.... viele weitere Theoreme

Dirac Spinoren: (4 Komponenten)

$E > 0$ Spinor: $u_{(p,s)}$ $(\not{p} - m)u_{(p,s)} = 0$

$\bar{u}_{(p,s)}(\not{p} - m) = 0$

mit adjungiertem Spinor: $\bar{u} = u^\dagger \gamma^0$

$E < 0$ Spinor: $v_{(p,s)}$ $(\not{p} + m)v_{(p,s)} = 0$

$\bar{v}_{(p,s)}(\not{p} + m) = 0$

Spin-Operator: $\vec{s} = \frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$

Helizitätsoperator: $\lambda = \vec{s} \cdot \frac{\vec{p}}{|\vec{p}|}$ Spin-Komponente parallel $\vec{p}$

Helizitäts-Eigenzustände: $u_{\lambda(p)}$ mit $\lambda = \pm 1$

Normierung: $\bar{u}_{\lambda(p)} u_{\sigma(p)} = 2m\delta_{\lambda\sigma}$

$\bar{v}_{\lambda(p)} v_{\sigma(p)} = -2m\delta_{\lambda\sigma}$

$\bar{u}_{\lambda(p)} v_{\sigma(p)} = 0 = \bar{v}_{\lambda(p)} u_{\sigma(p)}$

Vollständigkeits-Rel.: (4x4 Gl.) $\sum_\lambda u_{\lambda(p)} \bar{u}_{\lambda(p)} = \not{p} + m$

$\sum_\lambda v_{\lambda(p)} \bar{v}_{\lambda(p)} = \not{p} - m$

Spinordarstellung: Sei $\vec{p}$ entlang +z-Achse, Helizität $= \frac{\lambda}{2}$

$$u_{\lambda(p)} = \sqrt{E+m}\begin{pmatrix} \chi_\lambda \\ \frac{2\lambda|\vec{p}|}{E+m}\chi_\lambda \end{pmatrix}$$

mit $\chi_{+1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Anti-Teilchen: $v_{\lambda(p)} = -\lambda\gamma^5 u_{-\lambda(p)}$