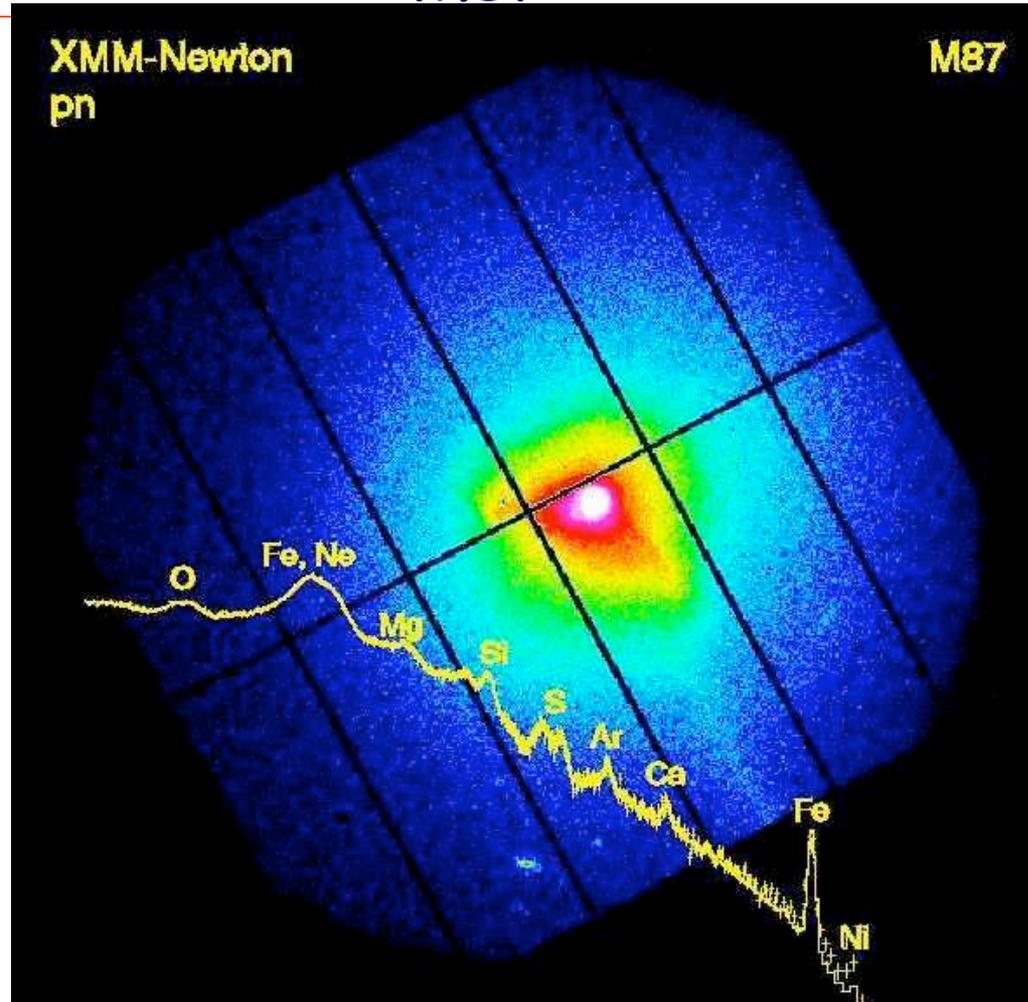


XMM Observations of the X-ray Halo of M87



Böhringer et al. 2001

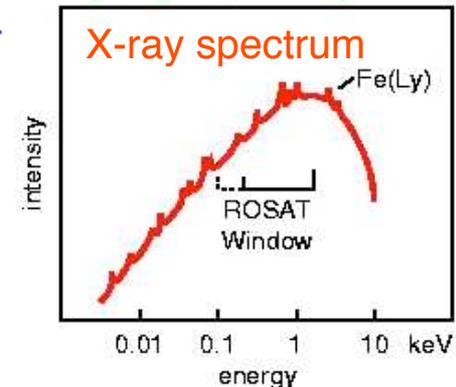
X-ray Observations of Galaxy Clusters

X-ray emission originates from 20-100 Mill. K plasma

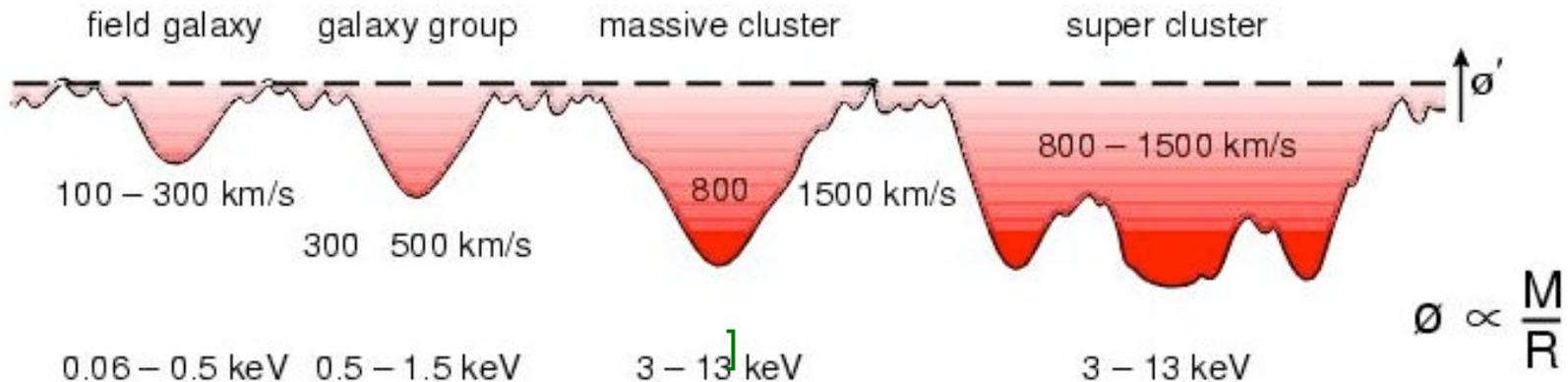
$$L_x = 10^{43} - 3 \cdot 10^{45} \text{ erg/s}$$

$$kT = 2-10 \text{ keV}$$

$$n_e \sim 10^{-4} - 10^{-1} \text{ cm}^{-3}$$



Sketch of the cosmic potential



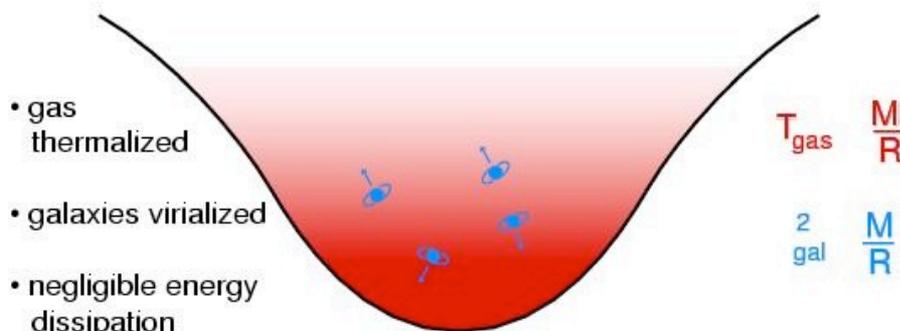
Comparison of galaxies and clusters as Dark Matter Halos

Galaxies



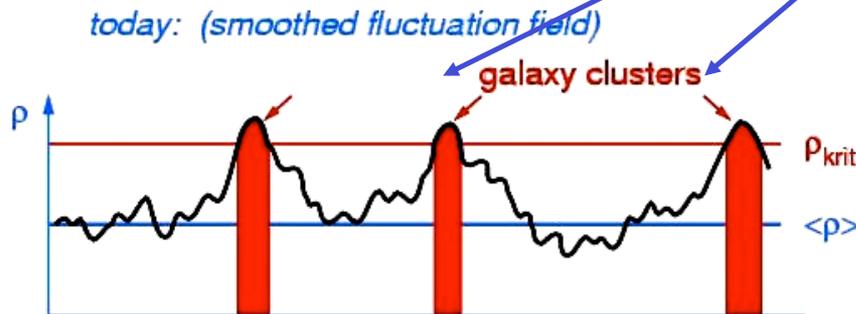
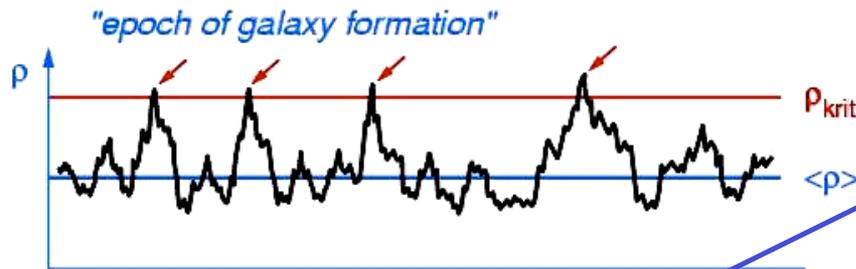
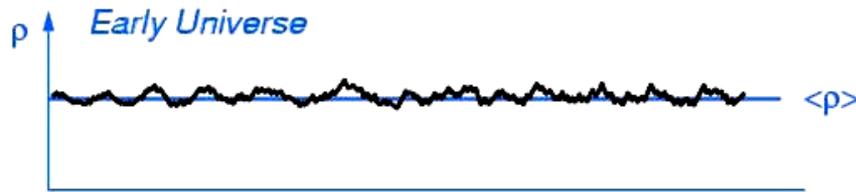
Complex relation between observable stellar population and dark matter halo

Clusters



The intracluster gas is heated when the cluster forms and does not cool – it still reflects the potential depth.

Formation of Galaxy Clusters as an Example of Collapsed Objects



Galaxy clusters (and analogously galaxies) are formed from density peaks in the density fluctuation field.

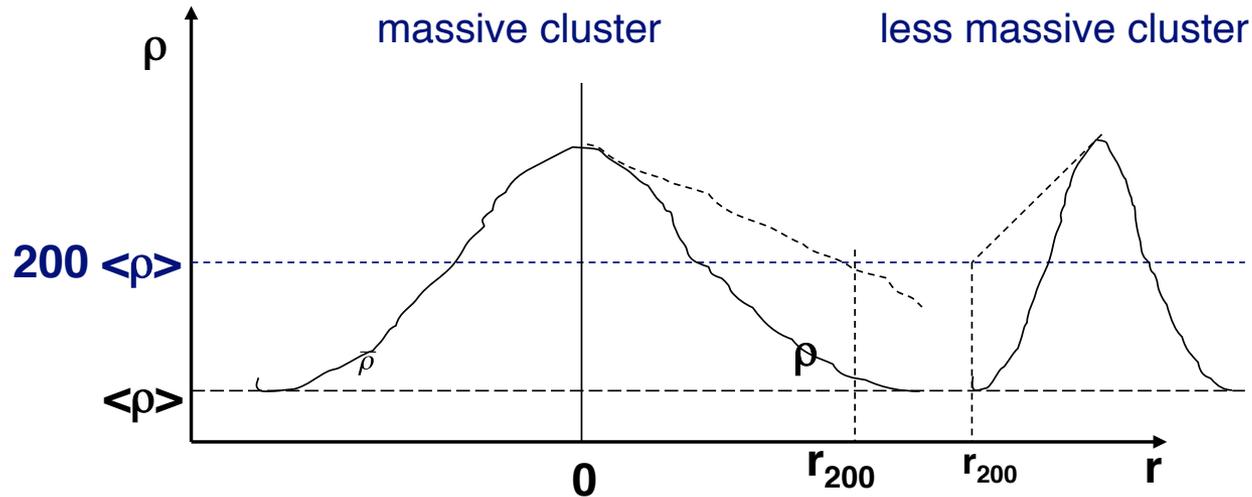
Galaxy clusters form part of the large-scale structure

Coma



Galaxy Counts from the Shane Virtanen Catalog (1957)

Definition of Fiducial („Virial“) Cluster Radius



Definition of fiducial radius, r_{200} :

$$M(r < r_{200}) \equiv 200 \bar{\rho} \frac{4\pi}{3} r_{200}^3$$

(e.g. NFW 1997 use r_{200} as fiducial (virial) radius)

Different Self-Similar Models

- All simulated cluster profiles have the same shape, independent of the halo mass, the initial fluctuation spectrum, cosmological parameters {NFW 97} and redshift. Thus there is a universal profile with some variation in the compactness parameter reflecting the „formation“ time.

Different forms of the proposed universal profile:

$$\rho(r) = \frac{\rho_s}{r/r_s (1+r/r_s)^3} \quad \text{Hernquist (1990)}$$

$$\rho(r) = \frac{\rho_s}{r/r_s (1+r/r_s)^2} \quad \text{NFW (1995)}$$

$$\rho(r) = \frac{\rho_s}{(r/r_s)^{1.5} [1+(r/r)^{1.5}]} \quad \text{Moore et al. (1999)}$$

$$M(r) = 4\pi\rho_s r_s^3 \left[\ln(1+c) - \frac{c}{1+c} \right] \quad (\text{NFW}); \quad M(r) = 4\pi\rho_s r_s^3 \frac{2\ln(1+c^{1.5})}{3} \quad (\text{M99})$$

$$\text{where: } c(\text{M99}) \approx (c(\text{NFW})/1.7)^{0.9}$$

X-ray Properties of Clusters of Galaxies

X-ray emission originates from thin, hot thermal plasma at temperatures of $\sim 20 - 100$ Mill. K

$$L_x = 10^{43} - 3 \cdot 10^{45} \text{ erg s}^{-1}$$

$$T_{\text{plasma}} = 2 - 15 \text{ keV}$$

$$R_x \sim 1 - 4.5 \text{ Mpc}$$

$$M_{\text{gas}} > M_{\text{gal}} \text{ (Fact. } \sim 2 - 5)$$

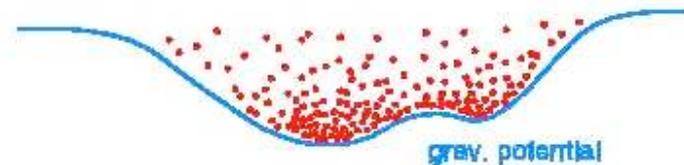
$$n_e \sim 10^{-4} - 10^{-1} \text{ cm}^{-3}$$

The plasma is primordially hot and cools only in the very center of some clusters - the temperature reflects the depth of the gravitational potential

Emission processes:

- Bremsstrahlung (free-free radiation)
- electronically excited emission line
- recombination radiation
- two-photon radiation

Plasma tracing gravitational potential



Hydrodynamic Structure of the Intracluster Medium

Hydrodynamic equations in spherical coordinates (spherical symmetric approx. of the cluster shape) - for stationary state :

Continuity equ.:
$$\frac{1}{r^2} \frac{d}{dr} (r^2 \rho v_r) = 0$$

Momentum equ. :
$$v \frac{dv}{dr} = -\frac{1}{\rho} \frac{dP}{dr} - \frac{GM(r)}{r^2}$$

(Euler equ.)

Energy equ. :
$$\frac{1}{r^2} \frac{d}{dr} \left[r^2 \rho v \left(\frac{1}{2} v^2 + \frac{5}{2} \frac{kT}{\mu m_p} + \Phi \right) \right] = -n_e n_H \Lambda(T) + H$$

For negligible kinetic pressure
$$\rho v \frac{dv}{dr} \approx 0$$

We get the hydrostatic equation:

$$\frac{1}{\rho} \frac{dP}{dr} = -\frac{GM(r)}{r^2}$$

Hydrostatic Approximation for the Mass Determination

$$\frac{1}{\rho} \frac{dP}{dr} = - \frac{GM(r)}{r^2} \quad P = nkT = \frac{\rho}{\mu m_p} kT$$

$$M(r) = - \frac{r^2}{G\rho} \frac{d}{dr} \left(\frac{\rho kT}{\mu m_p} \right)$$

$$M(r) = - \frac{r^2 k}{G\rho \mu m_p} \left(T \frac{d\rho}{dr} + \rho \frac{dT}{dr} \right)$$

$$M(r) = - \frac{r^2 k}{G \mu m_p} T \left(\frac{1}{\rho} \frac{d\rho}{dr} + \frac{1}{T} \frac{dT}{dr} \right)$$

$$M(r) = - \frac{kT r}{G \mu m_p} \left(\frac{d \ln \rho}{d \ln r} + \frac{d \ln T}{d \ln r} \right)$$

Note the analogy to the Jeans equation !

$$v^2 \cong \frac{kT}{\mu m_p}$$

The mass depends linearly on temperature, extent, and the relative density and temperature gradient

Significance of Clusters of Galaxies

- largest and most massive well defined objects in the Universe
 - size $\sim 10^{25}$ cm (3 Mpc)
 - mass $\sim 10^{14} - 10^{15}$ Sonnenmassen
- they contain 100s to 1000s of galaxies
- Clusters of galaxies are as interesting astrophysical laboratories and fundamental building blocks of our Universe as stars and galaxies – or even more exciting
 - They can be used in a similar way to probe the structure of our Universe as stars to probe the structure of our Milky Way

The β -Model II

We take the approximate solution for the central part of the King model :

Dark matter :
$$\rho = \rho_0 \left(1 + \left(\frac{r}{r_c} \right)^2 \right)^{-3/2}$$

X-ray plasma :
$$\rho_g = \rho_{g0} \left(1 + \left(\frac{r}{r_c} \right)^2 \right)^{-3/2\beta}$$

X-ray emission
measure :

$$\varepsilon = n_e^2 \tilde{\Lambda}(t) \quad n_e \propto \rho_g$$

Surface

brightness :

$$\begin{aligned} S_x(R) &= \frac{2}{4\pi} \int_R^\infty n_e^2 \tilde{\Lambda}(t) dz \\ &= \frac{2}{4\pi} \int_R^\infty \frac{n_e^2 \tilde{\Lambda}(t) r}{\sqrt{r^2 - R^2}} dr \end{aligned}$$

$$S_x(R) = \frac{n_{e0}^2 \tilde{\Lambda}(t) r_c}{4\pi} \frac{\Gamma(0.5)\Gamma(3\beta - 0.5)}{\Gamma(3\beta)} \left(1 + \left(\frac{R}{r_c} \right)^2 \right)^{-3\beta+1/2} (1+z)^{-4}$$

The β -Model III mass determination

From the last equation we can determine : $n_{e0} = f(S_{x0}, r_c, \tilde{\Lambda}(T), z)$

$$M(r) = -\frac{kT}{\mu m_p G} r \left(\frac{d \ln \rho_g}{d \ln r} \right)$$

$$\frac{d \ln \rho_g}{d \ln r} = -3\beta \frac{x^2}{1+x^2} \quad \text{with } x = \frac{r}{r_c}$$

$$M(r) = \frac{kT}{\mu m_p G} r_c 3\beta \frac{x^2}{1+x^2}$$

Which yields a convenient mass formula, often sufficient to obtain a mass estimate from observations.

Self-Similar Properties of Galaxy Clusters

- Since the cluster formation process (gravitational collapse) for the dark matter only is a self-similar process with no special scale and the dark matter dominates this process we can expect that clusters have a nearly self-similar appearance. This is interesting in particular for two reasons:
- Self-similarity in the cluster properties allow us to deduce all other cluster properties from the observation of a single global cluster parameter (e.g. X-ray luminosity).
- Deviations from the self-similar behaviour should be caused by extra physics introducing a special scale. Thus the study of these deviations will provide us with information about these extra physical processes.

Self-Similarity of Cluster Properties II

For the recent formation approximation we have then one fundamental relation between mass and cluster radius:

$$M_{\Delta} = \frac{4\pi}{3} r_{\Delta}^3 \rho_{crit} \Delta_c \quad \rho_{crit} = \frac{3H(z)^2}{8\pi G}$$

$$M_{\Delta} = \frac{1}{2} r_{\Delta}^3 \frac{\Delta_c}{G} H_z^2$$

$$r_{\Delta} = \left(\frac{2GM}{\Delta_c H_z^2} \right)^{1/3}$$

$$\text{SIS: } \rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

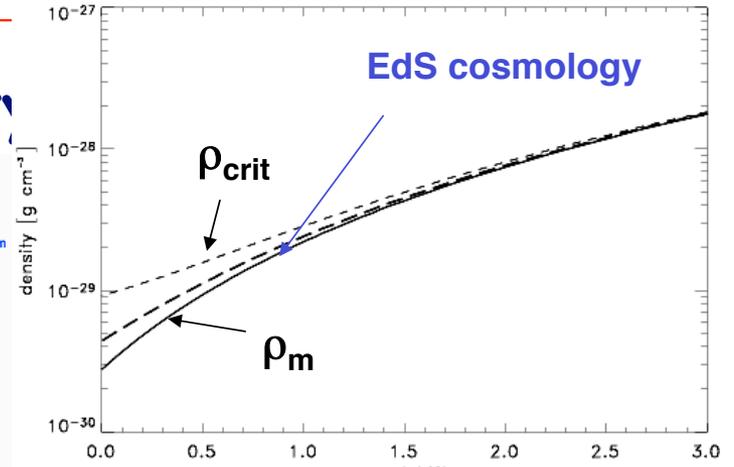
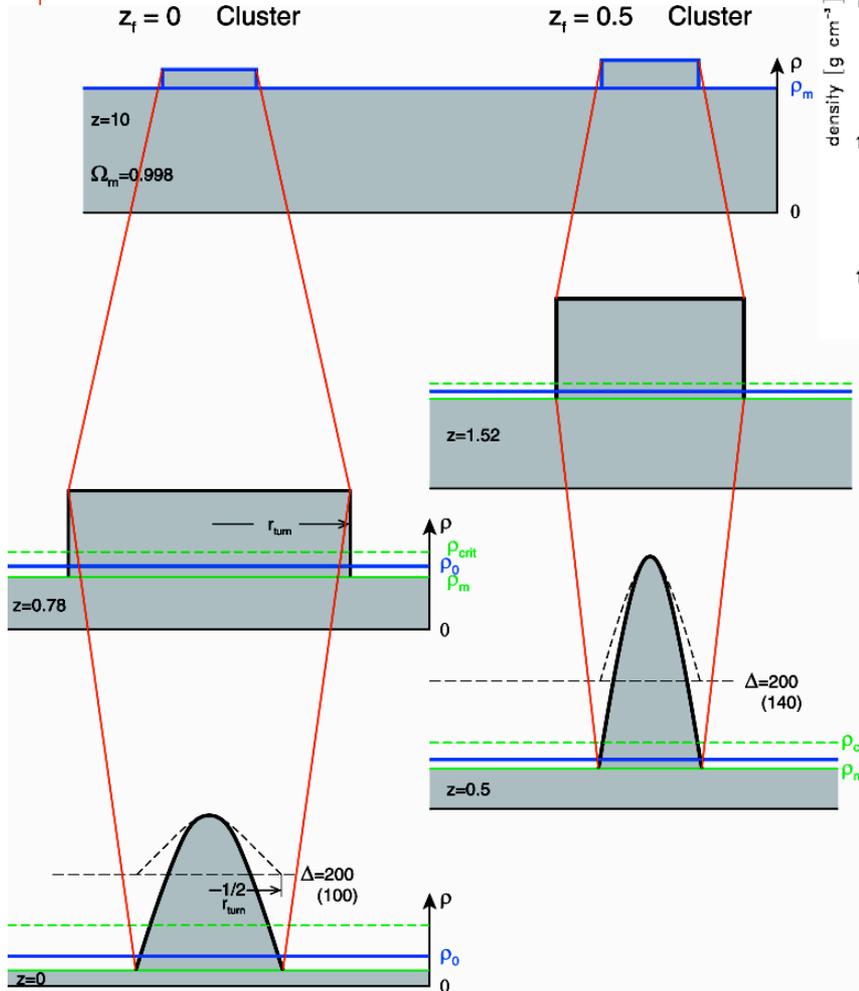
$$M_D = \sigma^2 \frac{4\pi}{2\pi G} \int_0^{r_{\Delta}} \frac{1}{r^2} r^2 dr = 2 \frac{\sigma^2}{G} r_{\Delta}$$

$$\sigma^2 = \frac{1}{2} \frac{GM_{\Delta}}{r_{\Delta}} = \left(\frac{1}{16} \right)^{1/3} (GM)^{2/3} (\Delta_c H_z^2)^{1/3} \quad E_z^2 = \frac{H_z^2}{H_0^2}$$

$$\sigma^2 = \left(\frac{1}{16} \right)^{1/3} (GH_0)^{2/3} M_{\Delta}^{2/3} F_z^{1/3} \quad E_z^2 = \Omega_m (1+z)^3 + \Omega_R (1+z)^2 + \Omega_{\Lambda}$$

$$\sigma^2 \propto M_{\Delta}^{2/3} F_z^{1/3} h^{2/3} \quad F_z = E_z^2 \Delta_c$$

Self-Similarity



$$r_{\Delta} \propto \left(\frac{M}{\Delta_c \rho_{crit}} \right)^{1/3}$$

$$\rho_{crit} = \frac{3 H^2}{8 \pi G} \propto H(z)^2$$

$$H(z)^2 = H_0^2 E(z)^2$$

$$\Rightarrow r_{\Delta} \propto \left[\Delta_c E(z)^2 \right]^{1/3} M^{1/3}$$

EdS → Concord.:

$$\Delta_c = 200 \quad \Delta_c = \Delta(z)$$